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Infrared radiation harmonic generation in degenerate semiconductors

G. Ferrante[1], M. Zarcone[1,a], and S.A. Uryupin[2]

[1] Dipartimento di Fisica e Tecnologie Relative, Università di Palermo and Istituto Nazionale per la Fisica della Materia, Viale delle Scienze, Edificio 18, 90128 Palermo, Italy
[2] P.N. Lebedev Physics Institute, Leninsky pr. 53, 119991, Moscow, Russia

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Abstract. The basic properties of infrared laser radiation odd harmonic generation due to electron-charged impurity collisions in degenerate semiconductors are investigated. It is found that in the case of relatively weak fields, the electron Fermi distribution is the cause of an anomalous suppression of harmonic radiation. In the case of strong fields, the effect of the selective suppression of single harmonics is established.

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1 Introduction

Investigation of infrared harmonic generation in semiconductors began many years ago (see, for example, [1,2]). It was rapidly established that, analogous to the situation occurring in a plasma [3], the nonlinear dependence of the electron collision frequency on the quiver velocity imparted to the electrons by a strong laser field acts as an important mechanism of high-order harmonic generation. In non-degenerate semiconductors, such a mechanism has been investigated for different collision processes: namely, for electron collisions with optical [4] and acoustic [5] phonons; and for electron collisions with charged impurities [6]. However, it soon became evident that it was extremely difficult to observe the harmonics generated via such a mechanism, and the subject lost its attractiveness.

For such a mechanism to act efficiently to generate harmonics, the collision frequency must be as large as possible. At the same time, a large collision frequency leads to efficient electron heating and, in some cases, to lattice heating as well. When it occurs, the electron nonlinear response is strongly damped, or the lattice destroyed. Recently new experimental possibilities have become available, which make it both timely and useful to reconsider and develop further the theory of harmonic generation in semiconducting media based on electron collisions. It is now possible to generate powerful ultrashort laser pulses, with durations comparable with the cycle corresponding to the laser fundamental frequency ω (see, for example, [7,8]). It is the aim of this Communication to undertake the theoretical consideration of infrared radiation harmonic generation in degenerate semiconductors based on electron-charged impurity collisions.

Below we derive the nonlinear correction to the current density due to collisions, when a laser radiation acts on a degenerate semiconductor. Such a correction allows us to find the electric field corresponding to the harmonic frequencies $(2n+1)\omega$, $n = 1, 2, ...$ Further, a general formula is derived for the harmonic generation efficiency. The generation efficiency is analyzed versus the ratio of the electron quiver energy imparted by the laser field $mv_E^2/2$ to the electron thermal energy kT, and versus the ratio of the chemical potential μ to kT. We report that, in a weak field, when the quiver energy is smaller than the chemical potential, in a degenerate semiconductor, harmonic generation is severely damped. The physical reason is traced back to the exponentially small number of electron free states below the Fermi level. For strong fields, when $mv_E^2/2$ is larger than μ by several times, we establish analytically and numerically the effect of disappearance of particular harmonics for specific values of the ratio $mv_E^2/2\mu$. Such an effect is not observed in non-degenerate semiconductors.

2 Harmonics of current density

The electron distribution over quasi-momenta in degenerate semiconductors is described by the Fermi function

$$f_F = \frac{2}{(2\pi\hbar)^3}\left\{\exp\left[\frac{\varepsilon(\overrightarrow{p})-\mu}{\kappa T}\right]+1\right\}^{-1}, \qquad (1)$$

where $\hbar$ is the Planck constant, κ the Boltzmann constant, T the electron temperature, μ the chemical potential, $\varepsilon(\overrightarrow{p})$ the electron energy as function of the

[a] e-mail: zarcone@unipa.it

quasi-momentum. In what follows we will restrict our analysis to semiconductors with the simplest dispersion law, i.e. $\varepsilon(\vec{p}) = p^2/2m$, where m is the electron effective mass. In formula (1) the chemical potential μ depends on temperature and electron density and is found from the relation

$$N = \int d\vec{p} f_F. \tag{2}$$

As we will consider highly degenerate semiconductors, when the Fermi energy $\varepsilon_F = \hbar^2(3\pi^2 N)^{2/3}/2m$ is much larger than the electron thermal energy κT, from (2) one has

$$\mu \simeq \varepsilon_F - \frac{\pi^2}{12}\frac{(\kappa T)^2}{\varepsilon_F}. \tag{3}$$

Let us consider the action of infrared laser radiation on such a semiconductor. We assume that the radiation frequency ω is much smaller than the forbidden band width, and much larger than both the electron plasma frequency ω_p and the effective collision frequency. The distance $2\pi v/\omega$ covered by an electron having velocity v in a laser field period is assumed to be small as compared with the field wavelength $\lambda = 2\pi/k$, where $\vec{k}$ is the wavevector. We note that in a strong laser field the characteristic electron velocity v may be larger than the Fermi velocity $v_F = \sqrt{2\varepsilon_F/m}$. We assume also that the electrons undergo collisions mainly with charged impurities, having charge e_i and concentration N_i. In such conditions, the response of the semiconductor electrons to the laser field action is determined by the kinetic equation

$$\frac{\partial f}{\partial t} + \frac{e}{m}\vec{E}\cos\left(\omega t - \vec{k}\vec{r}\right)\frac{\partial f}{\partial \vec{v}} = St(f), \tag{4}$$

where f is the non-equilibrium electron distribution function (EDF), $\vec{E}$ the laser field strength, e the electron charge, and $St(f)$ the electron-impurity collision integral. Taking into account that the electrons change slightly their quasi-momenta during collisions with charged impurities, the collision integral is written in the Fokker-Planck form

$$St(f) = \frac{1}{2}\nu(v)\frac{\partial}{\partial v_i}\left(v^2\delta_{ij} - v_i v_j\right)\frac{\partial}{\partial v_j}f, \tag{5}$$

where $\nu(v) = 4\pi e^2 e_i^2 N_i \Lambda \varepsilon_s^{-1} m^{-2} v^{-3}$ is the electron-impurity collision frequency, Λ the Coulomb logarithm, ε_s the dielectric permittivity, $v = p/m$. As $v/\omega \ll 1/k$ and $\omega \gg \omega_p$, equation (4) does not take into account the spatial variation of the field amplitude and of the EDF. The inequality $v/\omega \ll 1/k$ additionally allows to neglect the influence of the laser radiation magnetic field on the electron kinetics. As the collision frequency is much smaller than ω, in considering the electron high-frequency response, the collision integral in (4) may be taken into account according to perturbation theory. In the first approximation, neglecting the collisions in (4) we find

$$f_0 = f_F\left[\vec{v} - \vec{v}_E(t)\right], \quad \vec{v}_E(t) = \vec{v}_E \sin\left(\omega t - \vec{k}\vec{r}\right), \tag{6}$$

where $\vec{v}_E = e\vec{E}/m\omega$ is the electron quiver amplitude. The current density

$$\vec{j}_0 = e\int d\vec{p}\cdot\vec{v} f_0 = eN\vec{v}_E(t), \tag{7}$$

corresponds to the distribution (6). At frequencies larger than or of the same order as ω, the collision integral yields a small correction δf to the function f_0, $|\delta f| \ll f_0$. By integrating the equation for δf with the weight $e\vec{v}$ over the quasi-momenta one obtains the time derivative of the high-frequency part of the correction to the current density $\delta\vec{j} = e\int d\vec{p}\,\vec{v}\,\delta f$,

$$\frac{\partial}{\partial t}\delta\vec{j} = e\int d\vec{p}\cdot\vec{v}\, St\left\{f_F\left[\vec{v} - \vec{v}_E(t)\right]\right\}. \tag{8}$$

From (8), taking into account the collision integral explicit form (5) we obtain

$$\frac{\partial}{\partial t}\delta\vec{j} = ie\nu v_F^3 \int d\vec{p}\int\frac{d\vec{q}}{(2\pi)^3}\frac{4\pi}{q^2}\vec{q} \times \exp\left[i\vec{q}\vec{v} + i\vec{q}\vec{v}_E(t)\right] f_F(v), \tag{9}$$

where $\nu = \nu(v_F)$, and $f_F(v)$ is the Fermi EDF (1), $\vec{p} = m\vec{v}$. Considering that in (9) the exponent depends on time periodically through $\vec{v}_E(t)$, $\partial\delta\vec{j}/\partial t$ is written as a sum of the odd harmonics with frequencies $(2n+1)\omega$, where $n = 0, 1, ...$,

$$\frac{\partial}{\partial t}\delta\vec{j} = \sum_{n=0}^{\infty}\left(\frac{\partial}{\partial t}\delta\vec{j}\right)_{2n+1}\sin\left[(2n+1)(\omega t - \vec{k}\vec{r})\right]. \tag{10}$$

After integration over $\vec{q}$ and the quasi-momentum angles, for the amplitude of the $(2n+1)$ harmonic current density time derivative we obtain

$$\left(\frac{\partial}{\partial t}\delta\vec{j}\right)_{2n+1} = -16e\vec{v}_E\nu p_F^3\int_0^1 du\, u\, f_F(uv_E)F(n,u), \tag{11}$$

$$F(n,u) = \int_u^1\frac{dx\, x}{\sqrt{x^2-u^2}}\sin\left[(2n+1)\arcsin\left(\frac{u}{x}\right)\right]. \tag{12}$$

For the further analysis, the relation (11) is conveniently written as

$$\left(\frac{\partial}{\partial t}\delta\vec{j}\right)_{2n+1} = -eN\vec{v}_E\nu I(n,\gamma,\Delta), \tag{13}$$

where the dimensionless function $I(n,\gamma,\Delta)$ depends on the number n and on the parameters

$$\gamma = \sqrt{\frac{mv_E^2}{\kappa T}}, \quad \Delta = \frac{mv_E^2 - 2\mu}{2\kappa T}. \tag{14}$$

and has the form

$$I(n,\gamma,\Delta) = \frac{12}{\pi}\int_0^1 dt\, t\, F\left(n, \sqrt{1-t^2}\right) \times\left[1 + \exp\left(\Delta - \frac{1}{2}\gamma^2 t^2\right)\right]^{-1}. \tag{15}$$

3 Dependencies of current density in different cases

For the limiting cases of interest, the function $I(n,\gamma,\Delta)$ exhibits simple asymptotic forms. In particular, in the weak field limit, when

$$\mu \gg \kappa T \gg m v_E^2, \tag{16}$$

one has $\gamma \ll 1$ and $\Delta < 0$, $|\Delta| \gg 1$. With such γ and Δ from (15) one finds

$$I(0,\gamma,\Delta) \simeq 1 - e^{\Delta} \simeq 1 - \exp\left(-\frac{\mu}{\kappa T}\right), \tag{17}$$

$$\begin{aligned} I(n \neq 0,\gamma,\Delta) &\simeq \frac{3(-1)^{n-1}\gamma^{2n}}{8^n n!(2n+3)} e^{\Delta} \\ &\simeq \frac{3(-1)^{n-1}}{8^n n!(2n+3)} \left(\frac{m v_E^2}{\kappa T}\right)^n \exp\left(-\frac{\mu}{\kappa T}\right). \end{aligned} \tag{18}$$

The relations (13) and (17) describe the correction to the current density at the fundamental frequency ω due to collisions of electrons with charged impurities. It is just this correction which accounts for the laser field energy dissipation. In fact, using (7), (10) and (13) one obtains the absorbed power

$$\begin{aligned} Q &= \frac{\omega}{2\pi} \int_0^{\frac{2\pi}{\omega}} dt \left(\vec{j}_0 + \delta \vec{j}\right) \vec{E} \cos\left(\omega t - \vec{k}\vec{r}\right) \\ &= \nu \frac{E^2}{8\pi} \frac{\omega_p^2}{\omega^2} I(0,\gamma,\Delta), \end{aligned} \tag{19}$$

where $\omega_p^2 = 4\pi e^2 N/m$. From (17) and (19) follows that under the conditions (16) a small thermal spread in the Fermi distribution yields an exponentially small decrease of absorption.

The odd harmonics current density is given by formulae (13) and (18). According to (18), in the weak field case, odd harmonic generation is possible only thanks to the presence of the electron thermal motion. At $T \to 0$ harmonic generation does not take place. The harmonics current density amplitude dependence on n and on the field strength is similar to the case of a semiconductor exhibiting a maxwellian EDF, but contains the additional small exponential factor $\exp(-\mu/\kappa T)$ characterizing the electron distribution thermal spread near the Fermi level.

In a strong laser field the conditions may occur where

$$\mu \gg m v_E^2 \gg \kappa T. \tag{20}$$

In such a case $\gamma \gg 1$, but as before $\Delta < 0$, $|\Delta| \gg 1$. With such γ and Δ from (15) one finds

$$\begin{aligned} I(n,\gamma,\Delta) &\simeq \delta_{n,0} - (-1)^n \frac{12}{\sqrt{2\pi}} \gamma^{-3} e^{\Delta} = \\ &\delta_{n,0} - (-1)^n \frac{12}{\sqrt{2\pi}} \left(\frac{\kappa T}{m v_E^2}\right)^{3/2} \exp\left[\frac{m v_E^2 - 2\mu}{2\kappa T}\right], \end{aligned} \tag{21}$$

where $\delta_{n,0}$ is the Kronecker delta symbol. Rigorously speaking, for $n \gg 1$ the limiting transition from (18) to (21) occurs for γ values different from unity. However, for applications large n values are of small interest. As a matter of fact, for typical semiconductors, harmonics with $n \gg 1$ are efficiently absorbed due to interband transitions.

When the electron quiver energy is close to the Fermi energy, the following inequality may occur

$$\left|\frac{1}{2} m v_E^2 - \mu\right| \ll \kappa T, \tag{22}$$

which, jointly to $\mu \gg \kappa T$, gives $\gamma \gg 1$ and $|\Delta| \ll 1$. In such a case from (15) one has

$$\begin{aligned} I(n,\gamma,\Delta) &\simeq \delta_{n,0} - (-1)^n \frac{12}{\sqrt{2\pi}} \left(\sqrt{2}-1\right) \zeta\left(\frac{3}{2}\right) \gamma^{-3} = \\ &\delta_{n,0} - (-1)^n \frac{12}{\sqrt{2\pi}} \left(\sqrt{2}-1\right) \zeta\left(\frac{3}{2}\right) \left(\frac{\kappa T}{m v_E^2}\right)^{3/2}, \end{aligned} \tag{23}$$

where $\zeta(x)$ is the Riemann zeta function, $\zeta(3/2) \simeq 2.6$.

When the electron quiver energy satisfies the inequalities

$$\mu \gg \frac{1}{2} m v_E^2 - \mu \gg \kappa T, \tag{24}$$

for the parameters γ and Δ one has $\gamma \gg \sqrt{\Delta} \gg 1$, while formula (15) takes the form

$$\begin{aligned} I(n,\gamma,\Delta) &\simeq \delta_{n,0} - (-1)^n \frac{4}{\pi} \left(\frac{2\Delta}{\gamma^2}\right)^{3/2} \\ &= \delta_{n,0} - (-1)^n \frac{4}{\pi} \left(1 - \frac{2\mu}{m v_E^2}\right)^{3/2}. \end{aligned} \tag{25}$$

Finally, for very intense fields, when

$$\frac{1}{2} m v_E^2 \gg \mu \gg \kappa T, \tag{26}$$

$\Delta \simeq \gamma^2/2 \gg 1$, and from (15) one finds

$$\begin{aligned} I(n,\gamma,\Delta) &\simeq \frac{2}{\pi} \left(\frac{2\mu}{m v_E^2}\right)^{3/2} \\ &\times \left\{(2n+1)\left[\ln\left(\frac{2 m v_E^2}{\mu}\right) + \frac{2}{3}\right] - 2a(n)\right\}, \end{aligned} \tag{27}$$

where $a(0) = 0$, $a(1) = 4$, $a(2) = 28/3$, $a(3) = 232/15$, $a(4) = 776/35$. From the relations (13), (19) and (27) is seen that in a strong laser field the effective electron-impurity collision frequency is controlled by the electron quiver energy rather than by the Fermi energy. In other words, both the absorbed power and the harmonics current density are proportional to $\nu(v_E)$.

The approximate expressions (25) and (27) contain only implicitly the dependence on temperature, which manifests itself through the chemical potential. It permits the writing of a unique interpolation formula instead of (25) and (27). By substituting in (15) the function $[1+\exp(\Delta-\gamma^2t^2/2]^{-1}$ with the step-like Haviside function $\eta[\gamma_\mu^{-2}-1+t^2]$, where $\gamma_\mu^2 = mv_E^2/2\mu$, one has

$$I(n,\gamma_\mu) = \frac{12}{\pi}\int_0^{1/\gamma_\mu} du\, u\, F(n,u), \quad \gamma_\mu > 1. \tag{28}$$

Formula (28) may be integrated in closed form for arbitrary n. In particular, for relatively small n, having the largest interest, from (28) one finds

$$I(0,\gamma_\mu) = \frac{2}{\pi}\left[\frac{2}{\gamma_\mu^3}\ln\left(\gamma_\mu+\sqrt{\gamma_\mu^2-1}\right) + \arcsin\left(\frac{1}{\gamma_\mu}\right) - \frac{1}{\gamma_\mu^2}\sqrt{\gamma_\mu^2-1}\right], \tag{29}$$

$$I(1,\gamma_\mu) = \frac{12}{\pi}\gamma_\mu^{-3}\left[\ln\left(\gamma_\mu+\sqrt{\gamma_\mu^2-1}\right) - \sqrt{1-\gamma_\mu^{-2}}\right], \tag{30}$$

$$I(2,\gamma_\mu) = \frac{4}{3\pi}\gamma_\mu^{-3}\left[15\ln\left(\gamma_\mu+\sqrt{\gamma_\mu^2-1}\right) - \left(23-8\gamma_\mu^{-2}\right)\sqrt{1-\gamma_\mu^{-2}}\right], \tag{31}$$

$$I(3,\gamma_\mu) = \frac{4}{15\pi}\gamma_\mu^{-3}\left[105\ln\left(\gamma_\mu+\sqrt{\gamma_\mu^2-1}\right) - \left(197-164\gamma_\mu^{-2}+72\gamma_\mu^{-4}\right)\sqrt{1-\gamma_\mu^{-2}}\right]. \tag{32}$$

In the limiting cases (24) and (26), formulae (29–32) give the asymptotic expressions (25) and (27), respectively. Besides, for intermediate values of the laser intensity, when the transition from the inequality (24) to (26) takes place, the functions $I(n,\gamma_\mu)$ with $n \geq 2$ exhibit n-1 zeros. This property is seen also from Figure 1, which shows the dependencies of $I(n,\gamma,\Delta)$ on γ^2 for $n = 0,1,2,3$. The zeros at particular γ values are due to the vanishing of the corresponding terms of the series, in which the effective collision frequency has been expanded. We note that the dependencies reported in Figure 1 have been obtained by numerically integrating the expression (15), in which the finite value of the electron temperature is taken into account. The continuous curves correspond to $\mu/kT = 5$, while the dashed ones to $\mu/kT = 10$. We remark that the absence of some current harmonics at particular γ values is not a consequence of using the interpolation formula (28).

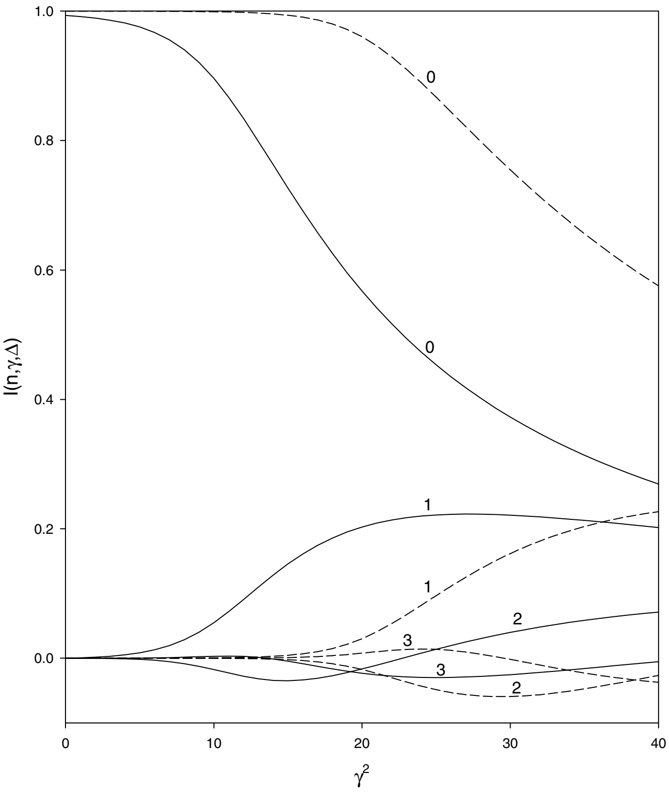

Fig. 1. Relative current density $I(n,\gamma,\Delta)$ at the frequencies $(2n+1)\omega$, $n = 1,2,3$ and current density dissipative component ($n = 0$) versus the ratio of the electron quiver energy to the thermal energy $\gamma^2 = mv_E^2/kT$. The continuous curve corresponds to a semiconductor for which the ratio of the chemical potential μ to kT is 5. The dashed curves correspond to $\mu/kT = 10$.

4 Efficiency of harmonic generation

The current density (13) allows us to find the harmonic field strength in the semiconductor point, where the source is located. From the Maxwell equations we have the wave equation, describing the harmonic field with frequency $(2n+1)\omega$,

$$\left(\varepsilon_h\frac{\partial^2}{\partial t^2} - c^2\Delta + \omega_p^2\right)\overrightarrow{E}_{2n+1}(\overrightarrow{r},t) = - 4\pi\left(\frac{\partial}{\partial t}\delta\overrightarrow{j}\right)_{2n+1}\sin\left[(2n+1)(\omega t - \overrightarrow{k}\,\overrightarrow{r})\right], \tag{33}$$

where c is the speed of light, ε_h the semiconductor dielectric permittivity in the infrared frequency domain. The forced solution to (33) has the form

$$\overrightarrow{E}_{2n+1}(\overrightarrow{r},t) = -\overrightarrow{E}_{2n+1}\sin\left[(2n+1)\left(\omega t - \overrightarrow{k}\,\overrightarrow{r}\right)\right]. \tag{34}$$

Taking into account the dispersion law of the fundamental wave $\omega^2 = (\omega_p^2 + k^2c^2)/\varepsilon_h$ and the relation (13), for the $(2n+1)$ harmonic field strength we find

$$\overrightarrow{E}_{2n+1} = \overrightarrow{E}\,\frac{\nu}{\omega}\,\frac{I(n,\gamma,\Delta)}{4n(n+1)}, \quad n \geq 1. \tag{35}$$

According to (34) and (35) the radiation at frequency $(2n+1)\omega$ propagates in the same direction as the fundamental wave and has the same linear polarization. The

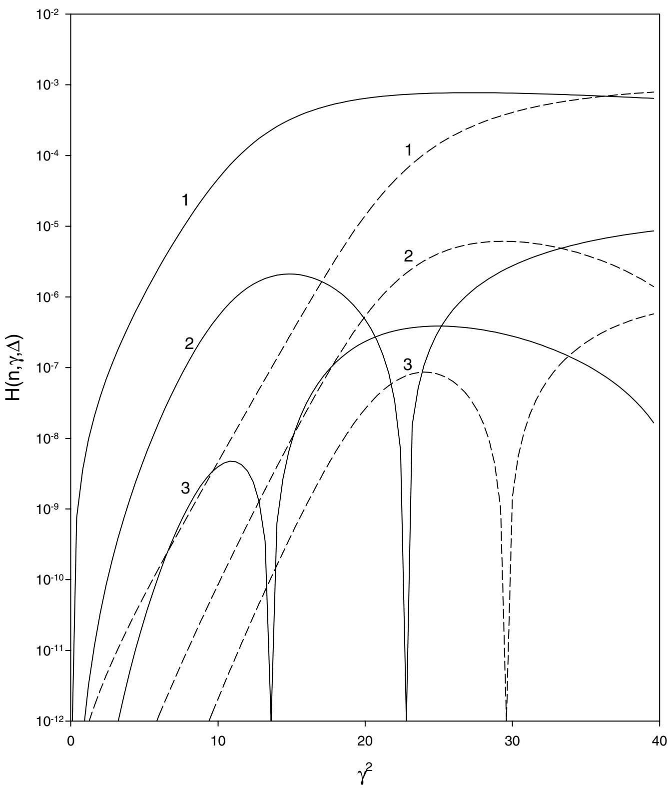

Fig. 2. Generation efficiency of the harmonics $(2n+1)\omega$, $n = 1, 2, 3$ versus $\gamma^2 = mv_E^2/kT$. Caption as for Figure 1.

ratio of the flux density at the frequency $(2n+1)\omega$, $I(2n+1) = cE_{2n+1}^2/8\pi$, to that of the fundamental wave, $I = cE^2/8\pi$, gives the odd harmonics generation efficiency

$$\eta(2n+1) = \frac{E_{2n+1}^2}{E^2} = \left(\frac{\nu}{\omega}\right)^2 \frac{I^2(n,\gamma,\Delta)}{16n^2(n+1)^2} \equiv \left(\frac{\nu}{\omega}\right)^2 H(n,\gamma,\Delta). \tag{36}$$

According to (36) the generation efficiency is proportional to the squared ratio of the collision frequency to the field fundamental frequency, while the non trivial dependencies on the harmonic number n, the radiation flux density, and the EDF degeneracy degree are buried in the function $H(n,\gamma,\Delta)$.

As $H(n,\gamma,\Delta) \sim I^2(n,\gamma,\Delta)$, the asymptotic expressions for $H(n,\gamma,\Delta)$ directly follows from (17), (18), (21), (23), (25) and (27). Figure 2 reports curves of the function $H(n,\gamma,\Delta)$ versus $\gamma^2 = mv_E^2/kT$ for $n = 1, 2, 3$. The continuous curves correspond to $\mu/kT = 5$, while the dashed ones to $\mu/kT = 10$. From Figure 2 one can appreciate how rapidly the harmonics generation efficiency grows in a weak field, by increasing the electron temperature. The curves with $\mu/kT = 5$ for small γ display significantly larger values. Besides, Figure 2 shows how abruptly the harmonic generation efficiency decreases in weak fields. In strong fields the dependency on γ^2 is smoother, however, there are isolated γ^2 values at which the generation of single harmonics does not take place.

Let us give an estimate of the third harmonics generation efficiency. Assuming $N = 10^{18}$ cm^{-3}, $m = 0.1m_0$, with m_0 the free electron mass, and $T = 77$ K, we have $\mu \approx \varepsilon_F \simeq 0.03$ eV and $\mu/kT \simeq 5$. If $N_i = 10^{16}$ cm^{-3}, $\Lambda/\varepsilon_s = 0.5$, $|e_i| = |e|$, the electron-impurity collision frequency in a weak field is $\nu \simeq 10^{13}$ s^{-1}. In such a situation, for a CO_2 laser radiation with $\omega \simeq 2\times10^{14}$ s^{-1} and flux density $I \simeq 2\times10^7$ W/cm^2 one has $\nu/\omega \simeq 5\times10^{-2}$ and $\gamma^2 \simeq 1$. As a result, from formula (36) and data shown in Figure 2, for the third harmonic generation efficiency one obtain $\eta(3\omega) \simeq 2.5\times10^{-11}$, which amounts to the flux density at 3ω $I(3\omega) \simeq 5\times10^{-4}$ W/cm^2. With the parameters considered in the present estimate, the characteristic time of the initial electron temperature doubling, due to laser energy absorption, is $\tau_T = 3NkT/2Q \simeq 3/\nu\gamma^2 \simeq 300$ fs. Accordingly, if the laser pulse duration is $\tau_p \leq 300$ fs, electron heating is negligible. If instead, $\tau_p > \tau_T$, insofar as $kT \ll \mu$, heating yields a relative increase of the generation efficiency thanks to the decrease of the electron degeneracy. If the temperature grows to the extent that $kT > \mu$, the generation efficiency diminishes due to the further decreasing of the small parameter γ^2.

In a more strong field, when $I = 10^9$ W/cm^2, one has $\gamma^2 = 50$, $\gamma_\mu^2 = 5$. In such a case, from (36) and Figure 2, $\eta(3\omega) \simeq 2.5\times10^{-6}$, $I(3\omega) \simeq 2.5\times10^3$ W/cm^2, and the temperature doubling time $\tau_T \approx 3/\nu\gamma^2 I(0,\gamma_\mu) \approx 2\pi/\omega \simeq 30$ fs is of the same order as the laser field period. As $\tau_p \gg 2\pi/\omega$, during the laser pulse action electron heating takes place. For instance, for $\tau_p = 10\pi/\omega \simeq 150$ fs, the initial temperature increases by a factor of 5. Such a temperature increase yields the removal of electron degeneracy, but the electrons still experience a strong field as effectively $\gamma^2 \geq 10$. As a result, the generation efficiency maintains the same order of magnitude as in the beginning of the laser pulse action. Numerical estimates are reported with the assumption that IR pulse duration is hundreds of femtoseconds. Presently the pulse durations of gas lasers are significantly longer, which makes it difficult to directly use such lasers to investigate harmonic generation. At the same time it is possible to use existing ultrashort pulses in the visible, to cut from a long IR pulse a part with the required duration. For instance, it may be done through rapid ionization of a dense matter with an ultrashort pulse in the path of the IR radiation propagation. With such a simple way to create a short pulse, a large part of the IR radiation energy is lost, but the energy flux density is left unchanged.

5 Conclusion

We have studied odd harmonic generation of infrared laser radiation in degenerate semiconductors in the process of electron scattering by charged impurities. It has been shown that degeneracy of electron distribution over velocities has a strong influence on the harmonic generation efficiency in the domain of not very high laser intensities. It is expected that similar influence takes place for other scattering mechanisms, now under consideration.

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